DOWNWARD SPATIALLY-SCALABLE IMAGE RECONSTRUCTION BASED ON COMPRESSED SENSING

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ABSTRACT

According to the compressed sensing (CS) theory, we can sample a sparse signal at a rate that is (much) lower than the required Nyquist rate, while still enabling a nearly exact reconstruction. Image signals are sparse when represented in a certain domain, and because of this, a large number of CS-based image sampling and reconstruction techniques have been developed recently. In this paper, we focus on the design of the downward spatially-scalable image reconstruction from the CS-sampled data. Traditional methods usually reconstruct an image whose size is the same as the original source image and then achieve the downward scalability through sub-sampling. In our proposed method, we unify these two steps into a single one and promise to deliver a much improved quality.

Index Terms—compressed sensing, spatial scalability, sparse representation

1. INTRODUCTION

With aid of the compressed sensing (CS) theory [1]-[2] – which has been developed for processing sparse signals over the past few years, we can now sample an original signal at a rate that is (much) lower than the required Nyquist rate and in a random fashion, while still enabling a nearly perfect reconstruction through solving a convex problem. There are two important aspects within the CS theory [3]. The first one is how to design the sampling mechanism. This includes how many samples to take; how to formulate these samples, i.e., using what kind of sampling matrix (purely random or with some structural properties); single pass or multiple passes for sampling; adaptive sampling; etc. The second one is how to perform the reconstruction so as to get the highest quality. It includes various reconstruction algorithms, such as the basis pursuit (BP) algorithm [4], the gradient projection for sparse reconstruction (GPSR) algorithm [5] and iterative thresholding (IT) algorithm [6].

Image signals may be sparsely represented in a certain domain. Therefore, more and more CS-based methods were proposed for the compression of image and video signals, including the block-based algorithms for image compression [7]-[8] and the distributed compressive video sensing [9]-[10]. At the front-end of a CS system, sensing usually takes place over a true scene in the real world (e.g., an airplane in sky). Clearly, the scene itself is a physical one and cannot be re-sized (neither up nor down). Nevertheless, we can only reconstruct a digital image from the received CS-sampled data. Here, the first issue one has to decide at the reconstruction stage is the resolution of the image to be reconstructed. Clearly, this depends on the displayer's size, and therefore spatially-scalable reconstruction appears as a topic – on which we have been focusing in our recent efforts. In this work, we simplify the above mentioned scenario as follows: the ground-truth imagery scene is also a digital image, but with a rather large resolution (to mimic a physical scene – whose resolution is infinity in theory). Therefore, we just need to consider the downward scalable reconstruction.

Traditional approach to obtain the downward scalability is as follows: an image with the same size as the original source image is first reconstructed, and then sub-sampled to achieve the required smaller size. However, when the original signal is a real scene in the physical world, it is impossible to reconstruct an image with the same size as the original scene. Moreover, even if the CS sampling is performed on an image with a limited size, the reconstructed image will be suffering from a poor visual quality when the CS-sampling rate is too low. This poor-quality image does not provide much help when a down-sizing is required to adapt to a specific display.

In this paper, we unify two steps mentioned in the above reconstruction (of the traditional approach) into a single one and try to directly reconstruct the down-sized image from the CS-sampled data. To achieve this goal, we propose two solutions: designing some specific sampling matrices or modifying the CS-sampled data, to reconstruct a high-quality down-sized image.

2. COMPRESSED SENSING AND SPATIALLY-SCALABLE IMAGE RECONSTRUCTION

Let's consider a 1-D ground-truth signal $\boldsymbol{x}_{1D}=[x_0,\dots x_{L-1}]^T\in\mathbf{R}^L$. In fact, a 2-D image signal $\boldsymbol{x}_{2D}=[x_{u,v}]_{H\times W}$ can be converted into a 1-D signal by the lexicographic re-ordering. To acquire the sampled data $\boldsymbol{y}_{1\text{D}}$, the CS theory proposes to apply a set of K linear functionals to $\boldsymbol{x}_{1\text{D}}$ to acquire the sampled data − which can be implemented via performing a random matrix on $\boldsymbol{x}_{1\text{D}}$ as follows:

$$\boldsymbol{y}_{1\text{D}}=\begin{bmatrix} y_0 \\ \vdots \\ y_{K-1}\end{bmatrix}=\begin{bmatrix} c_{0,0} & \cdots & c_{0,L-1} \\ \vdots & \ddots & \vdots \\ c_{K-1,0} & \cdots & c_{K-1,L-1}\end{bmatrix}\cdot\begin{bmatrix} x_0 \\ \vdots \\ x_{L-1}\end{bmatrix}=\boldsymbol{\Phi}\cdot\boldsymbol{x}_{1\text{D}} \tag{1}$$

Practically, $\boldsymbol{x}_{1\text{D}}$ is usually compressible in a certain domain; and how compressible is measured by the so-called sparsity. The CS theory tells that, for a highly sparse signal $\boldsymbol{x}_{1\text{D}}=[x_0,\ldots,x_{L-1}]^T\in\mathbf{R}^L$, we may take much fewer samples, i.e., $K<<L$, while still enabling a nearly exact reconstruction.

When performing the signal recovery, people always try to reconstruct a signal whose length (or size) is the same as that of the original one, i.e., to get $\hat{\boldsymbol{x}}_{1\text{D}}=[\hat{x}_0,\cdots,\hat{x}_{L-1}]\in\mathbf{R}^L$ (1-D case) or $\hat{\boldsymbol{x}}_{2\text{D}}=[\hat{x}_{u,v}]_{H\times W}$ (2-D case). Alternately, we consider in this work to reconstruct a signal whose length is *not* the same as that of the original one, i.e., to get $\boldsymbol{z}_{1\text{D}}=[z_0,\ldots,z_{l-1}]^T\in\mathbf{R}^l$, where l may be smaller or larger than L. In the 2-D case, we reconstruct an image of a difference size, i.e., $\boldsymbol{z}_{2\text{D}}=[z_{u,v}]_{h\times w}$ where h and w may be smaller or larger than H and W, respectively. We call this kind of reconstruction as the spatially-scalable reconstruction. Most typical selection in the 2-D case is to take $h=H/2^p$ and $w=W/2^p$ (downward scalability) or $h=2^pH$ and $w=2^pW$ (upward scalability), where p is an integer.

In practice, the real-world imagery scene may be very big; whereas the sampling rate (measured by the ratio K/L) is low due to reasons such as physical constraints, time-limitation, and dropping of badly-sampled data. Under this circumstance, it is no surprise that the reconstructed signal will be of an unaccepted quality. Meanwhile, it often happens that a display cannot support such a big size so that it is totally useless to reconstruct the original size. Therefore, it is highly necessary to reconstruct a high-quality image at a smaller size.

3. DOWNWARD SPATIALLY-SCALABLE IMAGE RECONSTRUCTION BASED ON CS

As a special case (but also an important one), let's assume $l=L/2$, i.e., we need to reconstruct a 2-fold down-sized signal. Mathematically, we need to reconstruct $\boldsymbol{z}_{1\text{D}}=[z_0,\ldots,z_{l-1}]^T\in\mathbf{R}^l$ to approximate the down-sized version of the ground-truth signal $\downarrow\boldsymbol{x}_{1\text{D}}=[x_0,x_2,\ldots,x_{L-2}]^T\in\mathbf{R}^{L/2}$.

Suppose that a total of K samples have been taken during the CS acquiring process. Although the ratio K/L is usually small, it will become 2 times larger if taking the ratio K/l when $l=L/2$. Furthermore, the ratio K/l goes to be even larger when l becomes smaller. In the 2-D case, 2-fold downsizing at two directions (vertical and horizontal) brings a factor of 4; while 4-fold downsizing gives a factor of 16. Thus, even with a pretty small sampling rate 10% applied on the ground-truth image, the rate corresponding to the down-sized image (by 2×2 or 4×4) becomes 40% (quite moderate) or 160% (oversampled!). Intuitively, people can anticipate that a high-quality image may be reconstructed with such a moderate to high sampling rate.

According to Eq. (1), we know that each CS-sampled data is obtained through a linear functional as:

$$y_i=c_{i,0}x_0+c_{i,1}x_1+\cdots+c_{i,L-1}x_{L-1}=\sum_{j=0}^{L-1}c_{i,j}x_j,\ i=0,\cdots,K-1. \tag{2}$$

If we assume that $x_{2j+1}\equiv x_{2j}\ \forall j$, Eq. (2) can be rewritten as:

$$\begin{aligned} y_i&=(c_{i,0}+c_{i,1})x_0+(c_{i,2}+c_{i,3})x_2+\cdots+(c_{i,L-2}+c_{i,L-1})x_{L-2}\\ &=\sum_{j=0}^{L/2-1}(c_{i,2j}+c_{i,2j+1})x_{2j}. \end{aligned} \tag{3}$$

Now, we have a very interesting observation: although each data y_i is originally acquired from the ground-truth signal $\boldsymbol{x}_{1\text{D}}=[x_0,\ldots,x_{L-1}]^T\in\mathbf{R}^L$, it can equivalently be regarded as the data that is CS-acquired directly from the 2-fold down-sized ground-truth signal $\downarrow\boldsymbol{x}_{1\text{D}}=[x_0,x_2,\ldots,x_{L-2}]^T\in\mathbf{R}^{L/2}$, while the random weighting coefficients become $c_{i,2j}+c_{i,2j+1}$. Since the sampling rate is no longer small with respect to this down-sized signal, it is no surprise that the reconstructed signal will end up with a good quality.

In the 2-D case, let's denote each CS-acquired data as:

$$y_i=\sum\left(\begin{bmatrix} c_{0,0}^{(i)} & \cdots & c_{0,W-1}^{(i)} \\ \vdots & \ddots & \vdots \\ c_{H-1,0}^{(i)} & \cdots & c_{H-1,W-1}^{(i)}\end{bmatrix}\otimes\begin{bmatrix} x_{0,0} & \cdots & x_{0,W-1} \\ \vdots & \ddots & \vdots \\ x_{H-1,0} & \cdots & x_{H-1,W-1}\end{bmatrix}\right),i=0,\cdots,K-1. \tag{4}$$

where ⊗ stands for dot-multiplication and ∑ takes the sum of all elements of a matrix. In the case of reconstructing a 2×2 down-sized image, let assume $x_{2u,2v}\equiv x_{2u+1,2v}\equiv x_{2u,2v+1}\equiv x_{2u+1,2v+1}$, Eq. (4) can be modified as:

$$\begin{aligned} y_i&=\sum\left(\begin{bmatrix} \begin{bmatrix} c_{0,0}^{(i)} & c_{0,1}^{(i)} \\ c_{1,0}^{(i)} & c_{1,1}^{(i)}\end{bmatrix} & \cdots & \begin{bmatrix} c_{0,W-2}^{(i)} & c_{0,W-1}^{(i)} \\ c_{1,W-2}^{(i)} & c_{1,W-1}^{(i)}\end{bmatrix} \\ \vdots & \ddots & \vdots \\ \begin{bmatrix} c_{H-2,0}^{(i)} & c_{H-2,1}^{(i)} \\ c_{H-1,0}^{(i)} & c_{H-1,1}^{(i)}\end{bmatrix} & \cdots & \begin{bmatrix} c_{H-2,W-2}^{(i)} & c_{H-2,W-1}^{(i)} \\ c_{H-1,W-2}^{(i)} & c_{H-1,W-1}^{(i)}\end{bmatrix}\end{bmatrix}\otimes\begin{bmatrix} \begin{bmatrix} x_{0,0} & x_{0,1} \\ x_{1,0} & x_{1,1}\end{bmatrix} & \cdots & \begin{bmatrix} x_{0,W-2} & x_{0,W-1} \\ x_{1,W-2} & x_{1,W-1}\end{bmatrix} \\ \vdots & \ddots & \vdots \\ \begin{bmatrix} x_{H-2,0} & x_{H-2,1} \\ x_{H-1,0} & x_{H-1,1}\end{bmatrix} & \cdots & \begin{bmatrix} x_{H-2,W-2} & x_{H-2,W-1} \\ x_{H-1,W-2} & x_{H-1,W-1}\end{bmatrix}\end{bmatrix}\right)\\ &=\sum\left(\begin{bmatrix} \sum\left(\begin{bmatrix} c_{0,0}^{(i)} & c_{0,1}^{(i)} \\ c_{1,0}^{(i)} & c_{1,1}^{(i)}\end{bmatrix}\right) & \cdots & \sum\left(\begin{bmatrix} c_{0,W-2}^{(i)} & c_{0,W-1}^{(i)} \\ c_{1,W-2}^{(i)} & c_{1,W-1}^{(i)}\end{bmatrix}\right) \\ \vdots & \ddots & \vdots \\ \sum\left(\begin{bmatrix} c_{H-2,0}^{(i)} & c_{H-2,1}^{(i)} \\ c_{H-1,0}^{(i)} & c_{H-1,1}^{(i)}\end{bmatrix}\right) & \cdots & \sum\left(\begin{bmatrix} c_{H-2,W-2}^{(i)} & c_{H-2,W-1}^{(i)} \\ c_{H-1,W-2}^{(i)} & c_{H-1,W-1}^{(i)}\end{bmatrix}\right)\end{bmatrix}\otimes\begin{bmatrix} x_{0,0} & \cdots & x_{0,W-2} \\ \vdots & \ddots & \vdots \\ x_{H-2,0} & \cdots & x_{H-2,W-2}\end{bmatrix}\right). \end{aligned} \tag{5}$$

Now, we see that, exactly the same as above, each data y_i can be regarded as the CS-acquired data directly from the 2×2 down-sized version of the ground-truth image $\downarrow x_{2\text{D}}$, while each random weighting coefficient becomes the sum of four elements within each 2×2 mask. Again, since the sampling rate becomes pretty high with respect to this down-sized image, the reconstructed image will end up with a very good quality.

However, it is never true in reality that $x_{2j+1}\equiv x_{2j}$ (in the 1-D case) or $x_{2u,2v}\equiv x_{2u+1,2v}\equiv x_{2u,2v+1}\equiv x_{2u+1,2v+1}$ (in the 2-D case). As

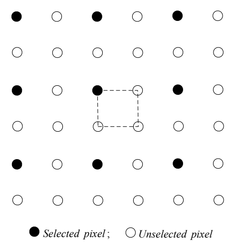

Fig.1. 2×2 down-sized pattern.

a consequence, the CS-acquired data $\{y_i\}$ do not represent the down-sized signal or image accurately; and such a representation could be very bad in some extreme cases. Based on these noise-corrupted CS data, no reconstruction can guarantee a good reconstruction quality. In the following, we propose two solutions to solve this unrealistic assumption.

3.1. Solution-A: Designing the CS-sampling matrix

In the first solution, some special weighting factors are arranged in the sampling matrix corresponding to the selected positions in the original image or image block. In this work, we focus on the 2×2 down-sized case, as shown in Fig.1. For this case, we can modify each $c^{(i)}_{u,v}$ by arranging special weighting factor for it as follows:

$$c'^{(i)}_{u,v} = f \cdot c^{(i)}_{u,v}, \quad (6)$$

where

$$f = \begin{cases} f_0 & \text{if } mod(u,2)=0 \text{ and } mod(v,2)=0 \\ f_1 & \text{otherwise} \end{cases} \quad (7)$$

and $f_0 \gg f_1$. With this specific sampling matrix (denoted as $\mathbf{\Phi}'$), although the assumption of $x_{2u,2v} \equiv x_{2u+1,2v} \equiv x_{2u,2v+1} \equiv x_{u+1,2v+1}$ is not holding, its negative impact on the reconstruction quality by using the CS-acquired data $\{y_i\}$ to represent the down-sized image tends to be decreased. In the extreme case, we can make f_1=0 so that this assumption would result no impact at all. Meanwhile, in order to adapt to the state-of-art reconstruction algorithm, such as the GPSR algorithm, the dimension of $\mathbf{\Phi}'$ must be reduced through adding the corresponding columns of it together according to the 2×2 down-sized pattern. This solution is much easier to be implemented by applying special weighting factors to some image pixels, but it won't be acceptable to most people as all image pixels in the locations $\{u,v\}$ which are not selected, i.e., $mod(u,2)\neq 0$ or $mod(v,2)\neq 0$, to compose the down-sized image are totally forgotten. To solve this problem, another solution which includes the information of all pixels is proposed next.

3.2. Solution-B: Modifying the CS-sampled data

The CS-sampled data $\{y_i\}$, i=0,…,K−1, are from the ground-truth image $x_{2D}=[x_{u,v}]_{H\times W}$ instead of the down-sized version $\downarrow x_{2D}$. As analyzed above, $\{y_i\}$ cannot be used directly to reconstruct a down-sized image. Then, our solution is to modify these data so that the new data $\{y'_i\}$ can represent more accurately the CS samples that are acquired directly from the down-sized image. To this end, we need a two-pass (or multiple-pass) approach. In the first pass, we reconstruct an image with the original size or follow the above assumption $x_{2u,2v} \equiv x_{2u+1,2v} \equiv x_{2u,2v+1} \equiv x_{2u+1,2v+1}$ to reconstruct the down-sized image. After this pass, we will perform some careful analysis on the reconstructed image to find out the inter-pixel relationships between pixels $x_{2u,2v}$, $x_{2u+1,2v}$, $x_{2u,2v+1}$, and $x_{2u+1,2v+1}$. These relationships will help us to modify the CS-sampled data $\{y_i\}$ so that a higher quality is expected in the next pass (of reconstruction).

In the 2×2 down-sized case, the details about the reconstruction algorithm are presented as follows:

1. ***First pass:*** we reconstruct an image $\hat{\boldsymbol{x}}_{2\mathrm{D}} = [\hat{x}_{u,v}]_{H\times W}$ with the original size.
2. ***Second pass:***

   Step-1: Based on the selected pixel, calculate the residual image $\boldsymbol{r}_{2\mathrm{D}} = [r_{u,v}]_{H\times W}$ as:

   $$\begin{cases} r_{k,l} = 0 \\ r_{k,l+1} = \hat{x}_{k,l} - \hat{x}_{k,l+1} \\ r_{k+1,l} = \hat{x}_{k,l} - \hat{x}_{k+1,l} \\ r_{k+1,l+1} = \hat{x}_{k,l} - \hat{x}_{k+1,l+1} \end{cases},$$

   where $\{k,l\}$ belongs to the selected positions as shown in Fig.1;

   Step-2: Perform the CS-sampling on $\boldsymbol{r}_{2D}$ to generate $\{\Delta y_i\}$;

   Step-3: Compensate $\{\Delta y_i\}$ to $\{y_i\}$ as: $y'_i = y_i + \Delta y_i$;

   Step-4: Generate $\hat{\mathbf{\Phi}}$ by reducing the dimension of $\mathbf{\Phi}$ via adding the corresponding columns of it together according to the 2×2 down-sized pattern;

   Step-5: Input $\{y'_i\}$ and $\hat{\mathbf{\Phi}}$ to an available reconstruction algorithm to compose a down-sized image.

3.3. Comparison between two solutions

Based on the above discussion, implementing the downward spatially-scalable reconstruction with Solution-A seems much easier due to the utilization of special weighting factors on the selected pixels in the CS-sampling. More specifically, using a specific sampling matrix in Solution-A makes a non-uniform sampling over the image pixels and generates special CS-acquired data, thus leads to an easier down-sized reconstruction. It also means that the application of Solution-A must depend on such a specific sampling matrix. This limits its application in a generic scalable reconstruction in which the CS-acquired data are normally generated by using a common sampling matrix instead of a special one. Compared with Solution-A, Solution-B offers a more general way to implement the scalable reconstruction and the data compensation involved in it makes it more flexible to recover

Table 1. PSNR(dB) of 2×2 down-sized images with Solution-A.

Sampling rate		0.2	0.3	0.4	0.5
Lena	**Traditional**	27.570	32.015	35.107	37.694
	Proposed	30.524	34.173	36.503	38.454
Fishingboat	**Traditional**	25.324	26.505	30.871	33.822
	Proposed	27.996	31.525	34.154	36.004
Goldhill	**Traditional**	27.299	28.721	33.870	35.892
	Proposed	29.548	32.824	35.340	37.273

(a) (b)

Fig.2. Portions of 2×2 down-sized image --"Lena" with sampling rate=0.2: (a) traditional method; (b) Solution-A.

Table 2. PSNR(dB) of 2×2 down-sized images with Solution-B.

Sampling rate		0.2	0.3	0.4	0.5
Lena	**Traditional**	26.375	28.556	30.324	31.891
	Proposed	27.344	29.525	31.318	32.819
Fishingboat	**Traditional**	24.014	25.939	27.990	29.621
	Proposed	24.670	26.715	28.798	30.353
Goldhill	**Traditional**	25.527	27.148	28.633	29.968
	Proposed	26.377	27.994	29.467	30.784

(a) (b)

Fig.3. Portions of 2×2 down-sized image --"Lena" with sampling rate=0.2: (a) traditional method; (b) Solution-B.

an image with any size (besides the 2×2 down-sized case) according to the requirement from the consumer side – which is more suitable for the practical application.

4. EXPERIMENTAL RESULTS

Some experimental results are presented in this section to show the performance of our proposed method with different solutions. The CS-sampling is applied to each 32×32 image block and the GPSR algorithm is used to reconstruct these blocks in both the traditional method (i.e., reconstructing the original-sized image first and then down-sampling it) and our proposed method.

We first apply our proposed method with Solution-A to reconstruct the 2×2 down-sized images. While designing the sampling matrix, we make $f_0=5f_1$. The traditional method and our proposed method are performed on still images (with the size of 512×512) of *Lena*, *Fishingboat* and *Goldhill*, respectively. A group of sampling rates is tested in our simulations. The PSNR for both methods is presented in Table 1. Meanwhile, the visual results are shown in Fig. 2. It is seen from Table 1 and Fig. 2 that our proposed method offers significant improvement over the traditional method.

Then, we perform the proposed approach with Solution-B to implement the downward spatially-scalable reconstruction for the 2×2 down-sized images. The objective and subjective experimental results are presented in Table 2 and Fig. 3, respectively. From Table 2 and Fig.3, we find that our proposed method yields not only a higher PSNR but also a better visual perception than the traditional approach.

Finally, it is also found from these results that Solutions A and B perform quite differently, so do both compared traditional approaches, due to using different sampling matrices as well as different processing strategies. More specifically, the specific sampling matrix used in Solution-A brings it a higher quality gain (over its compared traditional method) than Solution-B which uses a normal sampling matrix does. However, using the specific sampling matrix limits the application of Solution-A in the generic scalable reconstruction as we have mentioned in Section 3.3, while Solution-B is more suitable for the practical application.

5. CONCLUDING REMARKS

In this paper, we proposed a one-step method to implement the downward spatially-scalable image reconstruction from the received CS data. Two solutions are developed in our proposed method to achieve this goal and both perform better than the traditional method.

The future work is to find more efficient ways to classify the CS data into "independent" and "dependent" so as to remove all "dependent" ones when a bigger factor is performed for down-sizing, because the total number of CS-acquired data $\{y_i\}$ can easily become larger than the down-sized image's size, leading to the so-called oversampling. Meanwhile, based on the image reconstructed in the first pass (as described in Section 3.2), we need to classify pixels into "good" and "bad" ones so as to decide which ones are helpful for the analysis of inter-pixel relationships in the next pass. Some recently developed image quality assessment methods will be applied here for this purpose. During this classification, we will also consider the fact that different structural patterns produce more or less visual impact. Finally, we will combine this classified result into the aforementioned modification of the CS-sampled data.

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